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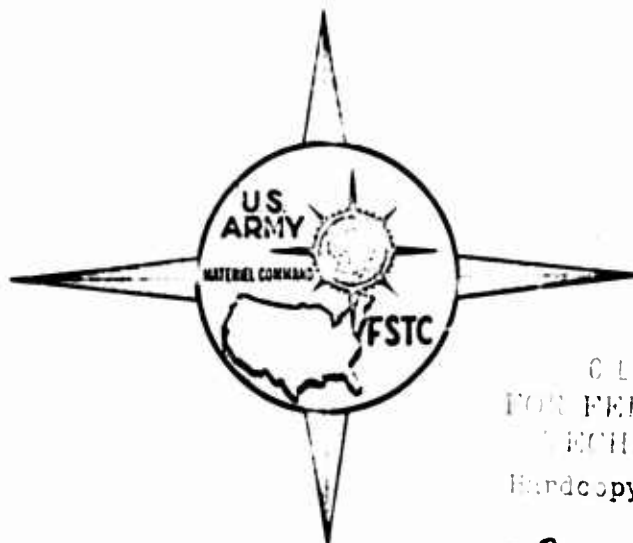
TRANSLATION

CERTAIN PROBLEMS IN THE DYNAMICS OF ESCAPEMENT REGULATORS WITHOUT NATURAL VIBRATIONS OF THE BALANCE

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MENT REGULATORS WITHOUT NATURAL VIBRATIONS
OF THE BALANCE

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Abstract: The article presents theoretical principles for calcu-
lating the vibration period of clock escapement regulators
without natural vibrations, taking into account the effect of
dynamic factors on the magnitude of the period. The formulas
obtained contribute to an understanding of certain laws govern-
ing the operation of these regulators (slight dependence of
the balance's vibration range upon the moment of the motive
force, the negligible influence of great changes in the moment
of inertia of the train upon the magnitude of the vibration
period etc.) and may be useful in the designing of clockworks
and the calculation of vibration periods.

CERTAIN PROBLEMS IN THE DYNAMICS OF ESCAPE-MENT REGULATORS WITHOUT NATURAL VIBRATIONS OF THE BALANCE

The article sets forth the principles of
the theory for calculating the vibration period
of regulators without natural vibrations, taking
into account the effect of dynamic factors on
the magnitude of the period. The dependences
here obtained contribute to an understanding of
the laws governing the operation of these regu-
lators and may be of use in designing and ana-
lyzing them.

Various branches of present-day engineering make ex-
tensive application of simplified clockwork using regulators
without natural vibrations (Figure 1).

Vibrations of the balance in systems with such regu-
lators take place exclusively by virtue of the interaction
of the teeth of the escape wheel with pallets of one form or
another, situated immediately on the balance or on a sepa-
rate anchor rigidly connected with the axis of the balance.
In character these vibrations are discontinuous auto-oscilla-
tions.

A balance has no fixed equilibrium position. There-
fore, amplitude is in significant measure an arbitrary param-
eter here, depending on where one's reckoning starts.

Only the double amplitude (range of vibrations) of
the balance is completely definite. It is equal to a com-
plete angle of rotation of the balance in one direction and
if the magnitude of amplitude has to be established, it can
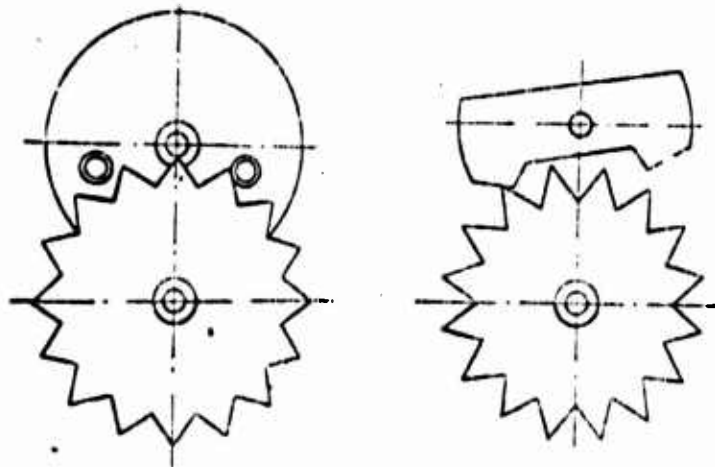be deemed equal to half this angle.

Figure 1. Diagrams of regulators with pin
pallets and flat pallets

The vibration range of a balance ordinarily does not
exceed 15-20°.

The chief parameter of the regulators here under con-
sideration is the vibration period. For a balance which is
not subject to the action of any restoring force the cycle
of motions performed by the balance between two adjacent
identical positions is said to be a complete vibration. The
vibration period determines the duration of this cycle. The
magnitude of the balance's vibration period depends consider-
ably upon the angular momentum on the escape wheel.

The theory here proposed for determining a balance's
vibration period and angles of rotation is based on the con-
cept of impulse angles depending not only on the design of
the escapement, but also upon moments of inertia of the bal-
ance and the entire wheelwork transmission, as well as the cor-
relations between moments on the anchor and the escape wheel
during impulses, and upon a number of other factors. It also
contains a method for finding the period and fundamental an-
gles with correction for the effect of the impacts of the
teeth of the escape wheel with the pallets on their contact
at the beginning of each impulse.

By taking these factors into account, we can explain
certain laws, here noted in practice, governing the operation
of these regulators: the slight dependence of the balance's
vibration range upon the moment of the motive force; the

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negligible influence of great changes in the moment of inertia
of the train upon the magnitude of the vibration period; etc.

The formulas obtained here make it possible to assess
the role of dynamic factors in the operation of regulators
without natural vibrations and to make a more valid selection
of design parameters in the design and planning of clockwork,
as well as to obtain more accurate results in the calculation
of vibration periods.

Positions of the Contact Points of Balance and Escape Wheel

In the vibrational steady state the pallets of the
balance meet with the teeth of the escape wheel at strictly
defined points, the positions of which on each pallet serve
as the reference point for the angles of rotation of the
balance during the operation of the regulator.

In order to find the angular coordinates of the ele-
ments of an escapement during contact, let us examine the
free rotation of the balance and wheel (Figures 2 and 3).

The beginning of free rotation coincides with the end
of an impulse, and the position of the escape wheel and the
balance at this moment is established by the appropriate plot-
ting or calculation.

Part of the angle of free rotation β_c, which we
shall designate $\beta_{cк}$, will be traversed by the wheel; the
other part, which we shall designate $\beta_{cб}$, will be trav-
ersed by the balance. Thus, the angle of free rotation of
the balance is

$$\beta_{cб} = \beta_c - \beta_{cк}. \tag{1}$$

[Translator's Note]: The subscripts here and here-
inafter are Russian letters. A summary list of sym-
bols with Russian-letter subscripts -- together with
expansion, translation and notation of first occur-
rence -- is given below at pages 22-24.

The second condition for finding the angles $\beta_{cб}$ and $\beta_{cк}$
is that the time during which the balance and the escape
wheel are in motion without contact is identical and can be

found not only from the equation of balance rotation but also from the equation of wheel rotation.

These equations have the form

$$I\ddot{\beta} - M_6 + M_\tau = 0, \tag{2}$$

$$I_x\ddot{\alpha} - M_x + M_{x6} = 0, \tag{3}$$

where β and α are the current angles of rotation of the balance and the escape wheel;

I and I_x are respectively the moment of inertia of the balance and that of the wheel transmission reduced to the axis of the escape wheel;

M_6 and M_{x6} are respectively the momentum acting upon the balance, and the momentum from the balance acting upon the escape wheel;

M_x is the angular momentum on the axis of the escape wheel;

M_T is the friction torque on the pivots of the balance.

In impulse sectors elements of the motion of the balance and the escape wheel are found by common solution of these equations. During free rotation the equations are solved separately with moments M_6 and M_{x6} equaling zero in this case.

Since in order to determine free-rotation time we must know the velocity of the balance and wheel at the end of the impulse, let us turn our attention to the common solution of equations (2) and (3).

If we say that

$$M_6 = M_{x6}\,\varphi(\beta),$$

we shall obtain

$$\left[I + I_x \frac{\ddot{\alpha}}{\ddot{\beta}}\,\varphi(\beta) \right]\ddot{\beta} - M_x\,\varphi(\beta) + M_\tau = 0.$$

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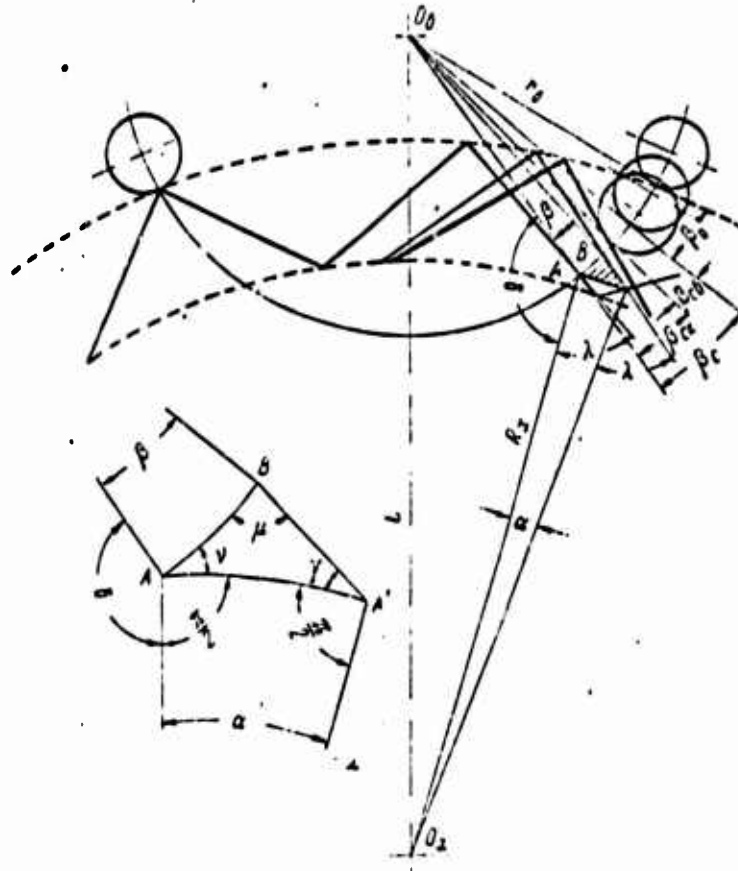
Figure 2. Diagram for determination of angles
of free rotation in case of pin pallets.

[Translator's Note]: The Russian subscript "б"
refers to the balance, "x" to the escape wheel.
For symbols with other Russian-letter subscripts
see summary list below at pages 22-24.

In view of the relative smallness of the impulse angles
for the regulators here under consideration, we shall assume
[1]

$$\frac{\ddot{\beta}}{\alpha} = \frac{\dot{\beta}}{\alpha} = i.$$ (4)

If in view of its poor variability throughout an
impulse we replace $\varphi(\beta)$ with mean value

$$\varphi(\beta)_{cp} = k,$$

Key: cp = mean

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we shall obtain the following equation of motion for the "balance-escape wheel" system

$$I_6 \ddot{\beta} - M_x k + M_\tau = 0,$$ (5)

where

$$I_6 = I + \frac{I_x k}{i}$$ (6)

is the reduced moment of inertia of the balance.

If we assume the velocity of the balance at the beginning of the impulse generally to be terminal velocity owing to post-impact reflection and equal to ψ_0, and if we integrate (5), we shall find its velocity at the end of the impulse

$$\psi_\text{H} = \sqrt{\frac{2(M_x k - M_\tau)}{I_6} \beta_\text{H} + \psi_0^2},$$ (7)

where β_H is angle of rotation of the balance during an impulse.

If we then consider free rotation and integrate equations (2) and (3) separately, we shall obtain expressions of the time of free motion.

From (2) given $M_T > 0$

$$t_c = \frac{I}{M_\tau} \left(\psi_\text{H} - \sqrt{\psi_\text{H}^2 - \frac{2 M_\tau}{I} \beta_{c6}} \right).$$ (8)

Given $M_\tau = 0$

$$t_c = \frac{\beta_{c6}}{\psi_\text{H}}.$$ (9)

From (3), given velocity of the escape wheel at the end of the impulse ξ_H ,

$$t_c = \frac{I_x}{M_x} \left(\sqrt{\frac{2 M_x}{I_x} \alpha_{c\kappa} + \xi_\text{H}^2} - \xi_\text{H} \right),$$ (10)

it being given that

$$\xi_\text{H} = \frac{\rho_{\text{H}}}{\rho_{x\text{H}}} \psi_\text{H} = \frac{\psi_\text{H}}{i_\kappa},$$ (10)

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where $r_{x_{II}}$ and $r_{6_{II}}$ are the distances from the centers of the escape wheel and the balance to the general normal to the surfaces of the impulses of the tooth and pallet at the moment when impulse transmission ceases, and $\alpha_{c\kappa}$ is the angle of free rotation of the escape wheel.

Let us establish the relation between angle of rotation of wheel α and angular displacement of impulse surface of tooth relative to axis of balance β.

From Figure 2 for escapement with pin pallets, if we disregard the curvature of arcs AB and AA' in view of the smallness of the angles of free rotation of the balance and the wheel, in considering $\triangle ABA'$ we shall have

$$AA' = R\alpha, \quad AB = r_6\beta \ \& \ \frac{r_6}{\sin \gamma} = \frac{R\alpha}{\sin \mu}. \tag{*}$$

From $\triangle O_x O_6 A$

$$\mu = \pi - (\nu + \gamma),$$

where

$$\gamma = \frac{\pi}{2} - \lambda; \ \nu = \pi - \delta; \ \delta = \pi - \text{arc cos} \ \frac{L^2 - R^2 - r_6^2}{2R r_6}.$$

[Translator's Note]: The subscript "6" refers to the balance.

Angles λ, γ and δ do not depend on the position of the escapement and that is why angle μ remains unchanged during free motion. Consequently, if we rewrite (*) in the form

$$\beta = \frac{R \sin \gamma}{r_6 \sin \mu} \alpha = D\alpha, \tag{12}$$

coefficient D will be constant and the sought relation between angles α and β will be linear.

If we reason similarly with reference to a balance with flat pallets on the anchor, it is easy to prove the correctness of dependence (12) in this case too, but angles γ and μ are more simply taken directly from the sketch of the escapement (Figure 3).

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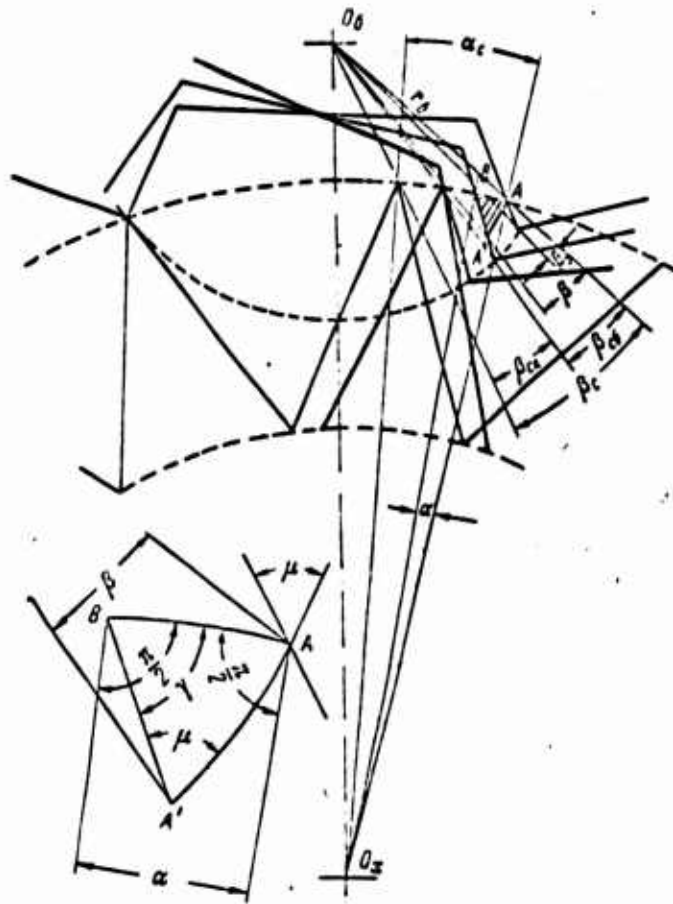
Figure 3. Diagram for determination of angles
of free rotation for flat pallets

[Translator's Note]: The Russian subscript "6"
refers to the balance, "x" to the escape wheel.
For symbols with other Russian-letter subscripts
see summary list below at pages 22-24.

If we substitute $\alpha_{cK} = \dfrac{\beta_{cK}}{D}$ into (10), if we

replace ξ_{μ} by Ψ_{μ} (11) and β_{cK} by β_{c6} in accordance
with (1), and if we equate expressions (8) and (10) given
$M_T > 0$, we shall obtain

$$\frac{I}{M_T}\left(1 - \sqrt{1 - \frac{2 M_T}{\Psi_{\mu}^2 I}\,\beta_{c6}}\right) = \frac{I_x}{I_x M_x}\left[\sqrt{\frac{2\,i_{\mu}^2 M_x}{\Psi_{\pi}^2 I_x D}(\beta_c - \beta_{c6}) + 1} - 1\right]. \quad (13)$$

If we equate (9) and (10) given $M_T = 0$, with allowance for the fact that

$$\beta_{\text{\tiny H}} = \beta_{c\delta} + \beta_{\text{\tiny H}}, \tag{14}$$

where β_0 (Figure 2) is the angle determining the position of the pallet at the beginning of free rotation and, further, with allowance for the fact that, given $M_T = 0$ and $\psi_0 = 0$, from (7)

$$\psi_{\text{\tiny H}} = \sqrt{-\frac{2k}{I_6}}\, \beta_{\text{\tiny H}}\, \sqrt{M_{\text{\tiny A}}} \tag{15}$$

we shall have

$$\beta_{c\delta} = \frac{2 I_{\text{\tiny K}} (\beta_{c\delta} + \beta_0) k}{I_6 I_{\text{\tiny K}}} \left[\sqrt{\frac{I_6 I_{\text{\tiny K}}^2 (\beta_c - \beta_{c\delta})}{I_{\text{\tiny K}} D k (\beta_{c\delta} + \beta_0)} + 1} - 1 \right]. \tag{16}$$

From the last equation it can be seen that in the absence of friction on the pivots of the balance and in the absence of balance repercussion during impacts with the wheel the position of the contact points of balance and escape wheel and, consequently, the magnitude of the balance's amplitude of vibrations will not be dependent upon the magnitude of the angular momentum on the escape wheel.

N. N. Bautin [2] also arrived at the same conclusion, using for his investigation the method of point transformations on a phase plane.

The friction on balance pivots and the velocities of the balance at the beginning of impulses ψ_0 are usually small. Therefore we can consider the above-indicated law to be characteristic in significant measure of all regulators of the type here under study.

If we solve (16) relative to $\beta_{\text{\tiny H}}$, we shall obtain the following expression for impulse angle

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$$\beta_\kappa = \frac{C_2 + \sqrt{C_2^2 - 4\,C_1\,C_3}}{2\,C_1},$$ (17)

where

$$C_1 = Da + 4D + 4\,i_\kappa; \quad C_2 = 2\,(Da\,\beta_0 + 2D\,\beta_0 + 2\,i_\kappa\,\beta_c + 2\,i_\kappa\,\beta_0);$$

$$C_3 = aD\,\beta_0^2 \ \& \ a = \frac{I\,i_\kappa}{I_\kappa\,k}.$$

In formula (17) only a plus stands in front of the root since, given $\beta_0 = 0$ and $C_3 = 0$, β_κ cannot equal zero.

Operational Regime of Regulators with Allowance for Impacts

Depending on the combination of different balance and escape-wheel parameters affecting impact results (magnitudes of moments of inertia, mechanical properties of materials, design factors etc.), the following post-impact states can be observed in regulators without natural vibrations after contact of the escape-wheel tooth with the pallet:

1) Withdrawal (recoil) of the wheel and rotation of the balance without reversal;

2) Recoil of the balance and wheel in opposite directions from the contact point;

3) Rotation of the wheel without reversal, and recoil of the balance.

As we enter upon a determination of the vibration period, we must establish the character of the operational regime [See Note] characteristic of a regulator with particular parameters. For this purpose the magnitudes and directions of the post-impact velocities of balance and escape wheel may serve as criteria.

[Note]: In principle different regimes may arise on each of the pallets.

Let us find the expressions determining these velocities.

Let the balance and wheel at the moment of contact have velocities of ψ_y and ζ_y respectively, and after impact ψ'_B and ζ_B respectively. Let us designate the distances from axes of rotation to impact line (Figure 4) ρ_i and ρ_x.

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and the total linear velocity of the colliding points at the
end of the first phase of impact U .
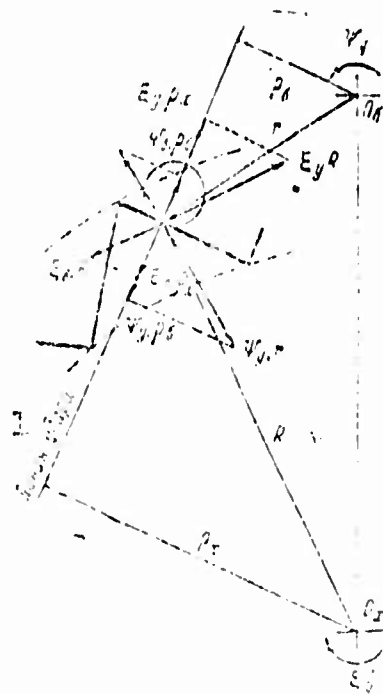
Figure 4. Diagram of "pallet-tooth" impact.

[Translator's Note]. The Russian subscript "б"
refers to the balance, "х" to the escape wheel.
For symbols with other Russian-letter subscripts
see summary list below at pages 22-24.

Key: 1. Line of impact

Let us take counterclockwise rotation as positive di-
rection. Then in the first of the above-indicated cases of
regulator operation we shall obtain the following pattern
of velocity transformation on impact:

$$\varepsilon_y \delta_b - U = -\varphi_n \rho_b;$$
$$-\varepsilon_y \delta_x - U = \varepsilon_n \rho_x.$$
$$(18)$$

If we assume the impact of tooth against pallet to be
elastic, in the first phase of impact (equalization of veloc-
ities) we shall have

$$I(U - \dot{\psi}_y \rho_6) = -S_1 \rho_6^2,$$
$$I_x(U + \xi_y \rho_x) = S_1 \rho_x^2.$$

For the second phase (restoration of velocities)

$$I(\rho_6 \dot{\psi}_6 + U) = -S_2 \rho_6^2,$$
$$I_x(\rho_x \xi_6 - U) = S_2 \rho_x^2.$$

If we eliminate from these equations first- and second-phase impulses S_1 and S_2 and velocity U, we shall obtain

$$\dot{\psi}_6 = (\xi_y + \xi_6) \frac{I_y}{I} \left(\frac{\rho_6}{\rho_x} \right) - \dot{\psi}_y. \qquad (19)$$

If we introduce coefficient of restitution of impact velocity

$$K = \frac{S_2}{S_1} = \frac{\rho_x \xi_6 + \rho_6 \dot{\psi}_6}{\rho_x \xi_y + \rho_6 \dot{\psi}_y} \qquad (20)$$

and by means thereof eliminate ξ_3 in (20), we shall find post-impact velocity of the balance

$$\dot{\psi}_6 = \frac{(1 + K) i \xi_y - \left(\frac{I}{I_x} i^2 - K \right) \dot{\psi}_y}{1 + \frac{I}{I_x} i^2}. \qquad (21)$$

According to the data of F. V. Drozdov [3], $K = 0.5$ to 0.6. If we know ψ_B, the post-impact velocity of the escape wheel can be computed according to the formula

$$\xi_6 = (\dot{\psi}_6 + \dot{\psi}_y) \frac{I}{I_x} i - \xi_y. \qquad (22)$$

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Analogous expressions for ψ_B and ξ_B are obtained also from a consideration of impact on the discharging pallet.

If ψ_B or ξ_B, computed according to (21) and (22), turn out to be negative, this will mean that the balance or escape wheel will not recoil from the contact point as assumed above, but will keep on moving in the pre-impact direction.

In establishing the operational regime of a regulator, or the beginning of an impulse, or initial velocity of a balance according to the velocities ψ_B and ξ_B for which magnitude and direction are known, we can be guided by the following considerations [See Note]:

[Note]: Let us disregard any possible subsequent impacts since they play no significant role in comparison with the first one.

1. If $\psi_B < 0$ and $\xi_B > 0$, impact does not reverse the rotation of the balance, but causes a withdrawal (recoil) of the escape wheel. After the angle of departure is passed, $\psi_0 = 0$.

The angle of departure is found from the circumstance that the post-impact kinetic energy of the wheel equals the work of the angular momentum at this angle, which yields

$$\alpha_{o\tau} = \frac{I_x \xi_{\text{\tiny B}}^2}{2 M_x} .\tag{23}$$

Key: OT [otkhod; departure]

The additional angle of rotation of the balance during withdrawal of the wheel in this case is

$$\beta_{o\tau} = \alpha_{o\tau} \, i,\tag{24}$$

Key: OT [otkhod; withdrawal, departure]

and the complete impulse angle is

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$$\beta_u = \beta_{c6} + \beta_u + \beta_{or}. \tag{25}$$

__2.__ Given $\psi_B > 0$ and $\xi_B > 0$, on-impact reflection of both balance and wheel takes place. The latter, having certain energy due to the magnitude of velocity ξ_B, initially withdraws at a certain angle (23) and then rotates, catching up with the pallet.

The extreme position assumed by the escape wheel on withdrawal can also be considered as the beginning of an impulse in this particular instance. The initial velocity of the balance must be taken approximately as $\psi_0 \approx \psi_B$.

__3.__ If $\dot{\psi}_B > 0$ and $\xi_B < 0$, only the balance will recoil from the impact point. The position of the parts of the escapement will correspond to the beginning of the impulse, $\psi_0 = \psi_B$.

The most frequently encountered regime characteristic of $I > I_x$ is the first.

From (21), (22) it can be seen that in order to compute ψ_B and ξ_B, we must know the pre-impact velocities of balance and escape wheel ψ_y and ξ_y which, in their turn, depend on $\dot{\psi}_0$ and ξ_0 and the impulse angles with correction for the effect of impacts.

ψ_y and ξ_y can be eliminated from (21), (22). However, in this case, complex biquadratic equations are obtained.

In practice it is more practical to find ψ_y and ξ_y by successive approximation according to the following procedure.

Assuming $\dot{\psi}_0 = 0$ at the beginning of the preceding impulse (the beginning determined without including the impact), we find the velocities of the balance and the escape wheel at the end of an impulse on one of the pallets and then their velocities at the moment of wheel-tooth contact with the next pallet in turn.

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Found in accordance with these velocities are the first approximate values of subsequent velocities, by taking which into account the impulse angle and velocity of wheel and balance on the preceding pallet etc. are refined until subsequent refinements no longer appreciably change results.

Let us note that owing to the degree of approximation of the classical impact theory used by us, as well as the inaccuracy and inconstancy of the coefficient of restitution K, it is quite sufficient to limit refinement to just one approximation.

Vibration Period of the Balance

If we designate impulse transmission time, free rotation time and the time required to pass through angle of departure respectively as t_u, t_c and t_{or}, according to the previously given determination of the vibration period we have

$$T = t_u' + t_u'' + t_c' + t_c'' + t_{or}' + t_{or}'' + 2 t_{y\text{д}}, \quad \text{[See Note]} \quad (26)$$

where $t_{y\text{д}}$ is the time the tooth is in contact with the pallet during impact, assumed to be identical on both pallets.

[Note]: Here and hereinafter the indexes ' and " designate quantities referring to receiving and discharging pallet respectively.

In view of the comparative smallness of the quantity $2 t_{y\text{д}}$ (approximately ten-thousandths of a second), given $T > 0.01$ sec. it is quite validly disregarded. In case of smaller values of the period, the duration of impacts can be allowed for by making experimental correction.

To find time intervals t_u, let us solve equation (5). By double integration we shall obtain

$$t_u = \frac{1}{C} (\dot{\varphi}_u - \dot{\varphi}_0),$$
(27)

where

$$C = \frac{M_k k - M_1}{I_0},$$
(28)

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and ψ_{M} is determined from (7).

Free rotation time is expressed by formulas (8) and (9).

If according to (21) and (22) we obtain $\psi_B < 0$ and $\xi_B > 0$, we shall find the duration of reverse rotation of the escape wheel from the equation

$$I_x \ddot{\alpha} + M_x = 0.$$

If we integrate under the initial conditions: $t = 0$; $\alpha = 0$ and $\xi = \xi_0$ and under the conditions at the end of withdrawal: $t = t_{OT}$; $\alpha = \alpha_{OT}$; $\xi = 0$ with allowance for (23), we obtain

$$t_{or} = \frac{I_x}{M_x} \xi_0. \tag{29}$$

Taking into account expressions (25), (27), (8) and (29), we shall obtain an expanded expression for determination of the vibration period of a balance without restoring force in the commonest case in the following form:

$$T = \frac{1}{C'}(\psi_{\text{M}}' - \psi_0') + \frac{1}{C''}(\psi_{\text{M}}'' - \psi_0'') + \frac{I}{M_{\text{T}}}\Big[\psi_{\text{M}}' + \psi_{\text{M}}'' - \Big(\sqrt{\psi_{\text{M}}'^2 - \frac{2M_{\text{T}}}{I}\beta_{c\delta}'} + \ldots$$
$$+ \sqrt{\psi_{\text{M}}''^2 - \frac{2M_{\text{T}}}{I}\beta_{c\delta}''}\Big)\Big] + \frac{I_x}{M_x}(\xi_0' + \xi_0'') + 2t_{y_1}, \tag{30}$$

where coefficients C' and C'' are found according to (28).

Given little friction on the balance pivots when one can assume $M_{\text{T}} = 0$,

$$T = \frac{1}{C'}(\psi_{\text{M}}' - \psi_0') + \frac{1}{C''}(\psi_{\text{M}}'' - \psi_0'') + \frac{\beta_{c\delta}'}{\psi_{\text{M}}'} + \frac{\beta_{c\delta}''}{\psi_{\text{M}}''} + \frac{I_x}{M_{\text{T}}}(\xi_0' + \xi_0'') + 2t_{y_1}. \tag{30a}$$

If we disregard friction and impacts,

$$T = \frac{A}{\sqrt{M_{\kappa}}}, \tag{30б}$$

where

$$A = \sqrt{T_6}\left(\sqrt[]{\frac{2\beta_u}{k'}} + \frac{\beta_{c6}}{\sqrt{2k'\beta_u'}}\right) + \sqrt{T_6}\left(\sqrt[]{\frac{2\beta_u}{k''}} + \frac{\beta_{c6}''}{\sqrt{2k''\beta_u''}}\right). \tag{31}$$

Considered by way of example is an escapement regulator without natural vibrations, having pin pallets, with the following data:

$l = 0,00015$ gram mm sec. squared; $\lambda' = \lambda'' = 56°$; $k' = 0,40$ [See Note];
$k'' = 0,39$; $\beta_c' = 8°18'$; $\beta_c'' = 12°29'$; $\beta_0' = 7°54'$; $\beta_0'' = 6°01'$; $i' = 1,75$; $i''' = 1,85$;
$i_\kappa' = 1,8$; $i_\kappa'' = 2,15$; $D' = 0,868$; $D'' = 0,923$; $a' = \frac{l}{l_\lambda}\cdot 5,658$; $a'' = \frac{l}{l_\lambda}\cdot 4,625$;
$M_\cdot = 0$; $K = 0,5$.

In view of the lack of necessary data the duration of impacts was not computed.

[Note]: Since in escapements with pin pallets, practically speaking, free rotation rather than impulse transmission takes place in sectors of the impulse along the pin, quantities k and i for these are determined only within the range of the impulse along the escape-wheel tooth.

Tables 1 and 2 present respectively the results of calculating the vibration period of a balance with and without allowance for the effect of impacts on vibrations. Table 3 shows the post-impact velocities of balance and escape wheel.

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Table 1

$\dfrac{1}{T_x}$	Входная палета		Выходная палета		A	T сек при M_x, гмм			
	$\delta'_м$	$I'_{б,}$ гмм сек²	$\delta''_м$	$I''_{б,}$ гмм сек²		4	25	100	200
10	10°44'	0,000153	9°29'	0,000153	0,02721	0,01360	0,00544	0,00272	0,00192
5	11°32'	0,000157	10°35'	0,000156	0,02843	0,01422	0,00569	0,00284	0,00201
2	12°28'	0,000167	12°02'	0,000166	0,03255	0,01628	0,00651	0,00326	0,00230
1	12°59'	0,000184	12°56'	0,000182	0,03591	0,01796	0,00718	0,00359	0,00254
0,5	13°24'	0,000219	13°34'	0,000213	0,03991	0,01996	0,00798	0,00399	0,00282
0,2	13°36'	0,000323	13°57'	0,000308	0,04917	0,02458	0,00983	0,00492	0,00348

[Translator's Note]: Symbols with Russian-
letter subscripts which have previously oc-
curred in text are listed below at pp. 22-24.

Keys:

1. Receiving pallet

2. Discharging pallet

3. T_{sec} given M_x [see summary list of symbols],
    gram-millimeters

4. gram-millimeters sec. squared

Table 2

$\dfrac{J}{T_x}$	1 Выходная палета			2 Выходная палета			3 T сек при M_x, гмм			
	$\alpha'_{от}$	$\beta'_{от}$	$\delta'_и$	$\alpha''_{от}$	$\beta''_{от}$	$\delta''_и$	4	25	100	200
10	2°40'	4°42'	15°26'	3°49	6°53	16°22	0,01751	0,00695	0,00314	0,00245
5	2°47'	4°52'	16°24'	3°51	6°55	17°30	0,01915	0,00753	0,00382	0,00265
2	2°52'	5°00'	17°28'	3°39	6°31	18°38	0,02159	0,00819	0,00434	0,00302
1	2°30'	4°22'	17°21'	2°54	5°14	15°10	0,02193	0,00851	0,00439	0,00303
0,5	1°24'	2°28'	15°52'	1°26	2°35	16°19	0,01820	0,00699	0,00359	0,00243
0,2	0°01'	0°02'	13°38'	0°00'	0°00'	13°57	0,01044	0,00435	0,00226	0,00122

[Translator's Note]: Symbols with Russian-letter subscripts which have previously occurred in text are listed below at pp. 22-24.

Keys:

1. Receiving pallet

2. Discharging pallet

3. T_{sec} given M_x [see summary list of symbols], gram-millimeters

Table 3

$\frac{l}{T_x}$	1 Входная палета				2 Выходная палета			
	3 $\psi'_в \frac{1}{сек}$ при M_x, гмм				4 $\psi''_в \frac{1}{сек}$ при M_x, гмм			
	2	25	100	200	4	25	100	200
10	−34,37	−85,93	−171,86	−239,85	−35,24	−88,11	−176,13	−249,01
5	−28,63	−71,54	−143,09	−202,26	−26,64	−66,58	−133,19	−188,34
2	−14,23	−35,63	−71,18	−100,66	−8,48	−21,22	−42,42	−59,98
1	3,52	8,84	17,60	39,64	11,84	29,56	59,19	83,73
0,2	25,35	63,39	126,90	179,86	35,88	89,73	179,35	250,99
0,2	58,17	124,63	249,14	352,38	61,25	153,28	306,55	432,99

$\frac{l}{T_x}$	5 $\xi'_в \frac{1}{сек}$ при M_x, гмм				6 $\xi''_в \frac{1}{сек}$ при M_x, гмм			
	4	25	100	200	4	25	100	200
10	157,98	204,45	789,00	1171,95	188,62	471,32	943,86	1334,56
5	113,79	180,15	569,16	804,98	133,98	335,03	669,99	947,04
2	72,96	115,36	364,82	515,87	82,39	205,95	411,97	582,46
1	48,18	76,20	240,98	366,40	52,02	130,03	260,05	367,74
0,5	25,72	40,27	128,70	181,95	25,84	64,59	129,18	180,27
0,2	1,73	2,74	8,68	12,30	−0,58	−1,37	−2,74	−4,08

[Translator's Note]: Symbols with Russian-
letter subscripts which have previously oc-
curred in text are listed below at pp. 22-24.

Keys:

1. Receiving pallet

2. Discharging pallet

3. $\psi'_в \frac{1}{sec}$ given M_x [see
   summary list of sym-
   bols], gram-mm

4. $\psi''_в \frac{1}{sec}$ given M_x [see
   summary list of sym-

bols], gram-mm

5. $\xi'_в \frac{1}{sec}$ given M_x [see
   summary list of sym-
   bols], gram-mm

6. $\xi''_в \frac{1}{sec}$ given M_x [see
   summary list of sym-
   bols], gram-mm

From the results presented we can draw the following basic conclusions.

1. Impacts and the resultant recoils of the escape wheel if $I \geqslant I_x$ cause an increase in the vibration period, but a decrease if $I < I_x$.

2. The vibration period of the balance is relatively slightly dependent upon the relation between the moment of inertia of the balance and the reduced moment of inertia of the escape wheel, with the greatest magnitude of the vibration period being obtained given $I \approx I_x$.

3. The vibration period of a balance without restoring force depends in great measure upon angular momentum.

4. The error in the determination of the period if impacts are disregarded amounts to (25-30%) given $I \geqslant I_x$, and may go as high as 50% or more given $I < I_x$. This must be borne in mind in designing clockworks with a bulky wheel system.

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SUMMARY LIST OF SYMBOLS WITH
RUSSIAN-LETTER SUBSCRIPTS.

Latin Symbols

Symbol & Subscript	Expansion & Translation of Subscript	Meaning of Symbol	First Occurrence
$I_б$	b [balans; balance]	Reduced moment of inertia of the balance	p. 6
$I_х$	kh [khodovoye koleso; escape wheel]	Moment of inertia of the train (wheel transmission) reduced to the axis of the escape wheel	p. 4
$M_б$	b [balans; balance]	Momentum acting upon the balance	p. 4
$M_т$	T [treniye; friction]	Friction torque	p. 4
$M_х$	kh [khodovoye koleso; escape wheel]	Angular momentum on axis of escape wheel	p. 4
$M_{хб}$	kh [khodovoye koleso; escape wheel] b [balans; balance]	Momentum from the balance acting upon the escape wheel	p. 4
$t_и$	i [impul's; impulse]	Impulse transmission time	p. 15
$t_{от}$	ot [otkhod; departure, withdrawal]	Time required to pass through angle of departure	p. 15
$t_с$	s [svobodnoye; free]	Free motion time; free rotation time	p. 6 p. 15
$T_{сек}$	sek [sekunda; second]	T_{sec}	p. 18
$t_{уд}$	ud [udar; impact]	Time the wheel tooth is in contact with pallet during impact	p. 15

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Greek Symbols

Symbol & Subscript	Expansion & Translation of Subscript	Meaning of Symbol	First Occurrence
α_{ot}	ot [otkhod; departure]	Angle of departure	p. 13
α_{ck}	s [svobodnyy; free] k [koleso; wheel]	Angle of free rotation of escape wheel	p. 7
β_{u}	i [impul's; impulse]	Angle of rotation of the balance during an impulse	p. 6
β_{ot}	ot [otkhod; withdrawal]	Angle of rotation of the balance during withdrawal of the escape wheel	p. 13
β_{c}	s [svobodnyy; free]	Angle of free rotation	p. 3
$\beta_{c\varepsilon}$	s [svobodnyy; free] b [balans; balance]	Part of angle of free rotation traveled by the balance	p. 3
β_{ck}	s [svobodnyy; free] k [koleso; wheel]	Part of angle of free rotation traveled by the escape wheel	p. 3
ξ_{δ}	v [vstrecha; contact, impact]	Velocity of escape wheel after contact with balance	p. 10
ξ_{u}	i [impul's; impulse]	Velocity of escape wheel at end of impulse	p. 6
ξ_{y}	u [udar; impact, contact]	Velocity of escape wheel at moment of contact with balance	p. 10

Symbol & Subscript	Expansion & Translation of Subscript	Meaning of Symbol	First Occurrence
ρ_b	b [balans; balance]	Distance from axis of rotation of balance to line of impact	p. 10
ρ_{bu}	b [balans; balance] i [impul's; impulse]	Distance from center of balance to the general normal to surfaces of impulses of tooth and pallet at the moment when impulse transmission ceases	p. 7
ρ_x	kh [khodovoye koleso; escape wheel]	Distance from axis of rotation of escape wheel to line of impact with balance	p. 10
ρ_{xu}	kh [khodovoye koleso; escape wheel] i [impul's; impulse]	Distance from center of wheel to the general normal to surfaces of impulses of tooth and pallet at the moment when impulse transmission ceases	p. 7
ψ_b	v [vstrecha; contact, impact]	Post-impact velocity of balance (after contact with escape wheel)	p. 10
ψ_u	i [impul's; impulse]	Velocity of balance at the end of impulse	p. 6
ψ_y	u [udar; impact, contact]	Velocity of balance at the moment of contact with wheel	p. 10